AVIRAL CLASSES
IIT-JEE | NEET | FOUNDATIONS

MATHEMATICS
CRASH COURSE

LECTURE-01

TOPICS : Trigonometry and Inverse Trig. Function

1. Number of solutions of the equation
tan x + sec x = 2 cos x, lying in the interval [0, 2π] is
(a) 1 (b) 2
(c) 3 (d) 0

2. If 0 ≤ x ≤ π and $81^{\sin^2 x} + 81^{\cos^2 x} = 30$ Then x is equal to
(a) π/3 (b) π/6
(c) 2π/3 (d) 5π/6

3. $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16}$ is equal to ?
(a) π/2 (b) 2π/5
(c) π/6 (d) π

4. If $x = \frac{1}{5}$, then the value of
cos ($\cos^{-1}$ x + 2$\sin^{-1}$x) is
(a) 2/5 (b) 1/5
(c) – 1/5 (d) – 2/5

5. The value of $\tan^{-1}$ 1 + $\tan^{-1}$ 2 + $\tan^{-1}$ 3 is
(a) 1 (b) 0
(c) π (d) – π

6. The value of $\cos^{-1}$ (cos12) – $\sin^{-1}$ (sin 12) is
(a) 0 (b) π
(c) 8 π – 24 (d) 8π – 32

7. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then $\cos^{-1}$ x + $\cos^{-1}$ y is equal to
(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

8. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then the value of
$$x^{100} + y^{100} + z^{100} - \frac{9}{x^{101} + y^{101} + z^{101}}$$
(a) – 1 (b) 0
(c) 1 (d) **2**

9. The value of $\cot^{-1}\left\{\frac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} - \sqrt{1+\sin x}}\right\}$ is
0 < x < 2π
(a) 2π – x (b) $\pi - \frac{x}{2}$
(c) $\frac{x}{2}$ (d) $2\pi - \frac{x}{2}$

10. If a_1, a_2, a_3,....a_n is an A.P. with common difference d, then
$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \ldots\ldots + \tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]$$
is equal to
(a) $\frac{nd}{1+a_1a_n}$
(b) $\frac{(n-1)d}{1+a_1a_n}$
(c) $\frac{(n-1)d}{a+a_n}$
(d) $\frac{a_n - a_1}{a_n + a_1}$

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CHEMISTRY
CRASH COURSE

LECTURE-01

TOPICS : Trigonometry and Inverse Trig. Function SOLUTION

1	B
2	A,B
3	D
4	C
5	C
6	C
7	A
8	B
9	B
10	--

1. (b) The given equation is meaningful only when $\cos x \neq 0$

i.e., $\sin x \neq 1$ or -1

Now, $\tan x + \sec x = 2\cos x \Rightarrow \frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$

$\Rightarrow \sin x + 1 = 2\cos^2 x \Rightarrow \sin x + 1 = 2(1 - \sin^2 x)$

$\Rightarrow 2\sin^2 x + \sin x - 1 = 0 \Rightarrow (1+\sin x)(2\sin x - 1) = 0$

$[\because \sin x \neq 1]$

$\Rightarrow \sin x = \frac{1}{2}$

$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$ in $[0, 2\pi]$.

2. (a, b) Let $81^{\sin^2 x} = y$. Then, $81^{\cos^2 x} = 81^{1-\sin^2 x} = \frac{81}{y}$

So that the given equation can be written as

$y^2 - 3y + 81 = 0 \Rightarrow y = 3$ or $y = 27$

$\Rightarrow 81^{\sin^2 x} = 3$ or $27 \Rightarrow 3^{4\sin^2 x} = 3^1$ or 3^3

$\Rightarrow 4\sin^2 x = 1$ or $3 \Rightarrow \sin^2 x = \frac{1}{4}$ or $\frac{3}{4}$

$\Rightarrow \sin x = \pm\frac{1}{2}$ or $\pm\frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{6}$ or $\frac{\pi}{3}$

3.
I. (d) $\sin^{-1}x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ and $\cos^{-1}x = \tan^{-1}\left(\frac{1-x^2}{x}\right)$

$$\therefore \quad \sin^{-1}\frac{12}{13} = \tan^{-1}\left[\frac{\frac{12}{13}}{\sqrt{\left(1-\frac{144}{169}\right)}}\right] = \tan^{-1}\left[\frac{\frac{12}{13}}{\frac{5}{13}}\right]$$

$$= \tan^{-1}\frac{12}{5}$$

$$\text{and} \quad \cos^{-1}\frac{4}{5} = \tan^{-1}\left[\frac{\sqrt{\left(1-\frac{16}{25}\right)}}{\frac{4}{5}}\right] = \tan^{-1}\left[\frac{\frac{3}{5}}{\frac{4}{5}}\right] = \tan^{-1}\frac{3}{4}$$

$$\therefore \quad \text{L.H.S.} = \left[\tan^{-1}\frac{12}{5} + \tan^{-1}\frac{3}{4}\right] + \tan^{-1}\frac{63}{16}$$

$$= \pi + \tan^{-1}\left[\frac{\left(\frac{12}{5}+\frac{3}{4}\right)}{\left(1-\frac{12}{5}\times\frac{3}{4}\right)}\right] + \tan^{-1}\frac{63}{16}$$

$$\left[\because \left(\frac{12}{5}\times\frac{3}{4}\right) > 1\right]$$

$$= \pi + \tan^{-1}\left(-\frac{63}{16}\right) + \tan^{-1}\frac{63}{16}$$

$$= \pi - \tan^{-1}\frac{63}{16} + \tan^{-1}\frac{63}{16} = \pi$$

4. (c). The given expression is equal to

$$\cos\left(\cos^{-1}x + \sin^{-1}x + \sin^{-1}x\right) = \cos\left(\frac{\pi}{2} + \sin^{-1}x\right)$$

$$= -\sin\left(\sin^{-1}x\right) = -x = -\frac{1}{5}.$$

5. (c) $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3$

$$= \tan^{-1}1 + \pi + \tan^{-1}\left(\frac{5}{-5}\right)$$

$$= \tan^{-1}1 + \pi + \tan^{-1}(-1) = \pi.$$

6. C

7. (a) $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$

$\Rightarrow \quad \frac{\pi}{2}-\cos^{-1}x+\frac{\pi}{2}-\cos^{-1}y=\frac{2\pi}{3}$

$\Rightarrow \quad \cos^{-1}x+\cos^{-1}y=\pi-\frac{2\pi}{3}=\frac{\pi}{3}.$

8. (b) We know that, $\left|\sin^{-1}x\right|\le\frac{\pi}{2}$

Therefore, $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$

$\Rightarrow \quad \sin^{-1}x=\sin^{-1}y=\sin^{-1}z=\frac{\pi}{2}$

$\Rightarrow \quad x=y=z=\sin\frac{\pi}{2}=1$

$\therefore \quad x^{100}+y^{100}+z^{100}-\frac{9}{x^{101}+y^{101}+z^{101}}$

$=3-\frac{9}{3}=0.$

9. (b) Since, $1\pm\sin x=\left[\cos\frac{x}{2}\pm\sin\frac{x}{2}\right]^2$

$\therefore \quad \cot^{-1}\left\{\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right\}$

$=\cot^{-1}\left\{\frac{\left[\cos\frac{x}{2}-\sin\frac{x}{2}\right]+\left[\cos\frac{x}{2}+\sin\frac{x}{2}\right]}{\left[\cos\frac{x}{2}-\sin\frac{x}{2}\right]-\left[\cos\frac{x}{2}+\sin\frac{x}{2}\right]}\right\}$

$=\cot^{-1}\left\{-\cot\frac{x}{2}\right\}=\cot^{-1}\left\{\cot\left(\pi-\frac{x}{2}\right)\right\}=\pi-\frac{x}{2}.$

10. (b) We have,

$\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+....+\tan^{-1}\left(\frac{d}{1+a_{n-1}}\right)$

$=\tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{a_3-a_2}{1+a_2a_3}\right)+...+\tan^{-1}\left(\frac{a_n-a_{n-1}}{1+a_{n-1}a_n}\right)$

$$= \left(\tan^{-1} a_2 - \tan^{-1} a_1\right) + \left(\tan^{-1} a_3 - \tan^{-1} a_2\right) + \ldots$$

$$\ldots. + \left(\tan^{-1} a_n - \tan^{-1} a_{n-1}\right)$$

$$= \tan^{-1} a_n - \tan^{-1} a_1 = \tan^{-1}\left(\frac{a_n - a_1}{1 + a_n a_1}\right)$$

$$= \tan^{-1}\left(\frac{(n-1)d}{1 + a_1 a_n}\right)$$

$$\therefore \quad \tan\left[\tan^{-1}\left(\frac{d}{1 + a_1 a_2}\right) + \tan^{-1}\left(\frac{d}{1 + a_2 a_3}\right) + \ldots\right.$$

$$\left. \ldots.. + \tan^{-1}\left(\frac{d}{1 + a_{n-1} a_n}\right)\right]$$

$$= \frac{(n-1)d}{1 + a_1 a_n}.$$